

CAUSALITY AND THE INDEPENDENCE PHENOMENON

The Case of the Demand for Money

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We examine the demand for money using causality results with data from two alternative policy regimes. For Spanish series of money and prices we obtain the same result of independence that Feige and others found with U.S. data. The result of the test for the German hyperinflation period reveals bidirectional causality. It is shown that the somehow striking results of widespread independence among economic time series do not disprove but rather confirm the existence of a true underlying causal relationship. Causality results, and independence in particular, give us testable restrictions for the structural form. In the case of models for expectations in the rate of inflation, these restrictions allow us to revalidate the stability of the demand for money as postulated by the Quantity Theory.

1. Introduction

This paper examines the implications of some results in time series analysis for structural economic modelling. It is a contribution to the so-called 'structural econometric modelling time series analysis' (SEMTSA) [Zellner (1979)]. We are concerned with the relationship between rationality, causality and feedback, and with the role of economic theory in modelling expectations. Our focus is on monetary theory and expectations about the rate of inflation, but the analysis has more general implications.

The relatively good forecasting performance of univariate time series models, as compared with large and complex econometric models [Cooper (1972) and Nelson (1972)], surprised the users of those models and stimulated some econometricians' interest in time series techniques. A reasonable conclusion to be drawn is that econometric models considered in the comparisons contain serious specification errors [Hickman (1972)], and thus relevant research should be channelled towards small systems of equations incorporating rational expectations.

As an alternative to the above, it might be tempting to conclude that for many economic variables it would be possible to substitute rational expectations by purely extrapolative ones. To anyone familiar with Granger's

causality as understood in time series analysis this second conclusion will, in turn, imply that for such economic variables, the most likely outcome of causality tests should be independence. And this independence has been actually found in many subsequent studies.

In particular Feige and Pearce (1976) have confirmed the lack of dependence between prices and money for the American economy. The implication that these authors see in their results is a need for a reformulation of the existing monetary theory.

With Spanish data we re-examine this question to find out whether these results conflict or not with the Quantity Theory of the demand for money.

In section 2 we briefly discuss the role of Box-Jenkins methods in the modelling of efficient extrapolative expectations which are, in any case, necessary to anticipate exogenous variables. In section 3 we intend to show that the equivalence of rational and extrapolative expectations may be 'an unfortunate and very special case' [Nelson (1975)], but not necessarily an unusual one. We pay special attention to those cases in which causality, as indicated by true economic laws, may be difficult to verify with tests based on Granger's predictive concept of causality. In section 4 we examine the observability problem of the natural rate hypotheses. This problem suggested by Sargent (1976) and discussed by Nelson, MacCallum and Sargent (1979) provides a good example of the more general case that we call inverse causality. We look for general methods to use causality tests to impose testable restrictions on proposed theories. We use this approach to construct a model of the influence of money on prices where this inverse causality appears. In section 5 we analyze series of money and prices of the Spanish economy. We model these series, perform tests of causality and verify the independence phenomenon. Using the results of section 4 and the model there constructed, we interpret this puzzling independence. In the final section we summarize the results and conclusions of the paper.

2. Expectations and efficiency

Expectations variables are needed in applied econometrics. Choice models of individual behaviour which lead to most economic theories are formulated in terms of perceived variables so that applications of such theories require specific assumptions about the way agents view the future. Directly observed expectations or anticipations are rare, hence implicit forecasting schemes are used.

Until recently, most models for expectations were of the type proposed by Cagan (1956) in his famous study on hyperinflation. They are linear extrapolative models: $\hat{y}_t(k) = \sum_{i=0}^{k-1} l_i(k) y_{t-i}$ where $\hat{y}_t(k)$ is the anticipated value from t for k periods ahead. One could hardly let statistical evidence suggest, what the l_i weights are, since almost always multicollinearity will be

present. For this reason parsimony is enforced by assuming a geometric decline in the weights. And some kind of error learning mechanism, such as $\hat{y}_t(1) - \hat{y}_{t-1}(1) = \alpha[y_t - \hat{y}_{t-1}(1)] = \alpha\hat{\varepsilon}_t$, from the prediction error $\hat{\varepsilon}_t$, is used to update the forecast. Muth (1961) showed the inefficiency of this adaptive scheme, unless the process generating the relevant time series is given by the model $y_t - y_{t-1} = \varepsilon_t - (1 - \alpha)\varepsilon_{t-1}$.

How can we propose a model for expectations that would be an efficient processor of the past of the observed time series? Without loss of generality, let us assume that our single time series is generated by the model $h(B)y_t = g(B)\varepsilon_t$ where $h(B)$ and $g(B)$ are polynomials in the lag operator B ($By_t \triangleq y_{t-1}$) of orders n and m , and from now on let $L_y(B) = g^{-1}(B)h(B)$ be the whitening filter for y_t ; ε_t is the corresponding innovation sequence. This is the representation extensively used by Box and Jenkins (1970) and many of their followers, for the case $\varepsilon_t \sim N(0, \sigma_t^2)$.

In this case the most efficient, one step ahead, causal predictor for time t will in fact satisfy $\hat{y}_t(1) = \arg \min E(y_{t+1} - \hat{y}_t(1))^2 = E(y_{t+1} | y_t, y_{t-1}, \dots, y_1)$ with $E =$ expected value operator, and can be derived from the estimated Box-Jenkins model as

$$\hat{y}_t(1) = -\hat{h}_1 y_t - \dots - \hat{h}_n y_{t-n} + \hat{g}_1 \hat{\varepsilon}_t + \dots + \hat{g}_m \hat{\varepsilon}_{t-m}. \tag{1}$$

We thus have a general method to find efficient extrapolations in the mean square prediction error sense. We first estimate the model generating the time series, and then form expectations using perhaps expression (1) or its equivalents.

It can be shown that this two-stage procedure can be collapsed into a one-stage procedure by using the self-tuner predictor

$$\hat{y}_t(l) = C(B)[h(B)f(B)]^{-1} \hat{\varepsilon}_{t-l}(l), \tag{2}$$

where the polynomials $C(B)$ and $f(B)$ are related to those of the true model of the series y_t , by the equation in B , $g(B) = h(B)f(B) + B^l C(B)$. Apart from other advantages of little relevance in this section [Hernández (1976)], expression (2) exhibits some very desirable properties as a model for expectations. It implies a learning by error of prediction, which is non-myopic although it is of finite memory. It certainly removes the ad hoc specification of the traditional expectations models.

3. Rationality, causality and theory

It is important to have expectations that are efficient extrapolations in the sense indicated in section 2. But rational agents may draw on an information set larger than just the past history of the variable being forecasted, including

the structure of the relevant system describing the economy [Muth (1961)]. Under which circumstances are efficient extrapolative expectations also rational?

Granger's concept of causality may be used to answer this question. According to Granger (1969), a variable y is not caused by the variable x if $\hat{y}_t(k) = E(y_{t+k}/A_t) = \arg \min E(\varepsilon_t(k)/A_t)^2 = E(y_{t+k}/A_t - x_t) = \arg \min E(\varepsilon_t(k)/A_t - x_t)^2$, where A_t represents all available information at time t and $A_t - x_t$ excludes the information about x at the same instant t . It is now clear that it is equivalent to say that y is 'caused' by x in the sense of Granger, or that a pure extrapolative model of expectations about y is irrational in the sense of Muth, or that y is endogenous in any model which includes x . This is the relationship between causality, rationality and endogeneity.

However, when Granger's definition is applied to test the existence of causality, it may be the case that for a given sample a variable x appears not to 'cause' another variable y , even though there is a law according to which y is influenced by x . Let us see some conditions under which this situation occurs.

Let the bivariate system be

$$\begin{bmatrix} P & Q \\ R & S \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}, \quad (3)$$

where $(e_{1t}, e_{2t})' \sim N(0, \Sigma)$, and P, Q, R, S are appropriate polynomials in B , which we can write as

$$\begin{aligned} y_t &= \tau(B)x_t + \xi_t = -RS^{-1}x_t + S^{-1}e_{2t}, \\ x_t &= F(B)y_t + \eta_t = -QP^{-1}y_t + P^{-1}e_{1t}, \end{aligned} \quad (4)$$

ξ_t and η_t are the disturbances; $\tau(B)$ is the transfer function and embodies a true theoretical relation or economic law; $F(B)$ is the feedback mechanism. The system (4) implies the following relationships between innovations:

$$\begin{aligned} \varepsilon_y &= L_y L_x^{-1} \tau(B) \varepsilon_x + L_y \xi_t, \\ \varepsilon_x &= L_x L_y^{-1} F(B) \varepsilon_y + L_x \eta_t, \end{aligned} \quad (5)$$

where L_x and L_y are the whitening filters for x and y .

- (i) If it happens to be true that $L_y L_x^{-1} \tau(B) = L_y S^{-1} = \gamma$ (constant), the innovations will be related by $\varepsilon_y = \gamma \varepsilon_x + u$. There would be no causality in the sense of Granger, even though x really causes y according to theory. A pure extrapolative model for expectations on y will be both

efficient and rational, and the set A_t [x_1, x_2, \dots, x_t and relations (4)] are of no value in predicting y .

- (ii) If we have a noiseless feedback $\eta_t \equiv 0$, we get $x_t = F(B)L_y^{-1}\varepsilon_y$, and because the filters to get the innovations are unique, we may write $\gamma\varepsilon_x = \varepsilon_y$ and $\gamma F(B) = L_y L_x^{-1}$. Again we face a situation in which the result of the test will be of no value to directly specify $\tau(B)$, and a rational agent would predict no better than an efficient extrapolator.
- (iii) More generally, whenever model (3) can be reduced to a triangular structure, through a similarity transformation, one of the variables becomes exogenous. For instance, if in the simplest case $R = P$ we can write

$$\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} P & Q \\ R & S \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} P & Q \\ 0 & Q - S \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} e_{1t} \\ e_{1t} - e_{2t} \end{bmatrix}, \tag{6}$$

the result of the test would be exogeneity for y_t .

Prior knowledge about the true model is, in those cases, essential to interpret correctly the results of the test. The above cases, although of a simple algebraic nature, are of great importance since they raise the problem of observational equivalence of alternative economic theories, as we will show in the following sections. Furthermore they indicate that equivalence between extrapolative and rational expectations occurs frequently for relevant economic variables.

4. Applications to monetary analysis

In the situations examined in the previous section, there is no equivalence between causality in the sense of Granger and causality in the sense of economic theory. This does not mean that the tests are uninformative about the structure of the underlying economic model. On the contrary, in these somehow special cases, causality analysis gives us testable restrictions on proposed models and may help to specify a given theory.

In the spirit of Zellner and Palm (1974) we can use the univariate Box–Jenkins models and the cross-correlations between innovations to check the structural specification of economic models. We point out two interesting situations:

4.1. Absence of causality

If we obtain $r_{\varepsilon_x \varepsilon_y}(k) = 0$ for all $k \neq 0$ and $r_{\varepsilon_x \varepsilon_y}(0) \neq 0$, we may write

$$\varepsilon_y = \gamma\varepsilon_x + e_2, \quad \gamma = r(0) \frac{\sigma_{\varepsilon_x}}{\sigma_{\varepsilon_x}}$$

and therefore obtain the system

$$\dot{y}_t = \gamma L_x L_y^{-1} x_t + L_y^{-1} e_{2t} \quad \text{and} \quad x_t = L_x^{-1} \varepsilon_x, \quad (7)$$

where we have written x conventionally as the exogenous variable. Thus the result of the test imposes the transfer function restriction, $\tau(B) = \gamma L_x L_y^{-1}$.

This is the situation examined by Nelson and McCallum (1979) when they discuss the observability of the classical model that we can write as $L_y y_t = \gamma(m_t - E_{t+1} m_t) + u_t$, where m_t is nominal money stock and y_t is real GNP. If we add $L_x m_t = \varepsilon_{t,m}$, we immediately get $\tau(B) = L_x L_y^{-1}$. Thus, the restriction from our analysis is the same as the identifying condition of the classical model. A naive reading of the causality test would be that m_t does not cause y_t , i.e., has no predictive value to anticipate y_t . Yet we can use the analysis to identify the model, proving the existence of a stable causal relation from m_t to y_t .

The stability issue is of crucial interest, and for this reason we would like to extend the analysis to other monetary policy regimes than that above, $L_x m_t = \varepsilon_{t,m}$. McCallum (1979) claims that the argument in favour of the observability of the natural rate hypothesis carries through to the case of a non-deterministic feedback. This is true as we will show.

4.2. Inverse causality

For the case of a general monetary policy under feedback we can write the classical model in the form

$$\begin{bmatrix} d(B) & v(B) \\ 0 & L_y \end{bmatrix} \begin{bmatrix} m_t \\ y_t \end{bmatrix} = \begin{bmatrix} e_{1,t} \\ u_t + \gamma e_{1,t} \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 = \varepsilon_y \end{bmatrix}. \quad (8)$$

It is evident from (8) that, through causality tests, we will get influence from y_t to m_t but absence of causality from m_t to y_t . In this sense we speak of inverse causality. When we read the test in a predictive sense only, we get the wrong diagnosis of the true theoretical influence of m_t on y_t . In the light of the analysis in section 3, we see that all that the classical hypothesis implies is that the system is triangular, i.e., money has no value to anticipate y_t . In the particular case discussed by Nelson the system is diagonal. The argument about the observability of the natural rate hypothesis remains valid for any general feedback rule.

In the general case of system (8), causality analysis immediately gives us the filter L_y of y_t and the rest of the model follows from the innovations cross-correlations. Since $\varepsilon_m = H(B)\varepsilon_y + K(B)e_1$ (innovations regression), we get $d(B) = L_m K^{-1}$, with L_m the filter of m_t , and $d^{-1}(B)v(B) = F(B) = H(B)L_m^{-1}L_y$. Thus the test gives us the testable restriction for the classical model under any feedback policy regime.

We show next that a similar situation arises when we specify a system of demand and supply of money to interpret the series of rates of inflation and of money creation.

We may write the demand for money according to the quantity theory as

$$\ln M_t - \ln P_t + \alpha J_t = u_t, \tag{9}$$

where M is the stock of money, P the price level, and J the expected rate of inflation. We assume that changes in real income and real interest rates are relatively unimportant to be able to use a bivariate framework of analysis. We, of course, include a disturbance term u_t in the transfer function, and let $u_t - u_{t-1}$ be white noise, so that we can work with first differences.

Muth's rationality allows us to write

$$J_t = \frac{1}{1 + \alpha} \sum_{i=1}^{\infty} \left(\frac{\alpha}{1 + \alpha} \right)^{i-1} \hat{x}_t(i), \tag{10}$$

where $\hat{x}_t(i) = (1 - B)M_t(i)$.

Note that the model of expectations about the rate of inflation depends on what is going to be the money supply process, which enters in J_t through $\hat{x}_t(i)$ the expectations about the theoretically exogenous variable. In particular, if we let the money supply become endogenous using the feedback rule,

$$x_t = (1 - \lambda)y_t / (1 - \lambda B) + e_{1t} \quad \text{and} \quad y_t = (1 - B)\ln P_t, \tag{11}$$

we get the same model for the expected rate of inflation that was used by Cagan, namely the adaptive scheme

$$J_t = (1 - \lambda)y_t / (1 - \lambda B). \tag{12}$$

In this situation we can write the bivariate model as

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} 1 & (1 - \lambda)/(1 - B) \\ 0 & (1 - \lambda B)/(1 - B) \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}. \tag{13}$$

Now, the univariate model for x_t is

$$(1 - B)x_t = (1 - \beta B)\varepsilon_x,$$

with

$$\beta = \{\sigma_{e_1}^2 + (1 - \lambda)\text{cov}(e_1, e_2)\} / \sigma_{e_x}^2, \tag{14}$$

and the relationship between innovations,

$$\varepsilon_x = (1 - \lambda)\varepsilon_y / (1 - \beta B) + (1 - B)e_1 / (1 - \beta B). \tag{15}$$

Therefore we have a triangular system, with the rate of inflation exogenous in the sense of Granger. We can estimate λ , the supply mechanism. However, the parameter α of the demand equation is not observable.

5. Analysis of the series of money and prices: The independence phenomenon and the quantity theory

We apply our previous analysis to the system described above, using two different sets of data. According to our model there is a stable demand for money which is the fundamental hypothesis to be eventually refuted, and there is a supply mechanism which is not necessarily invariable under different circumstances. So, we examine evidence from different data sources in order to test the stability of the demand function when the supply mechanism changes.

5.1. Spanish data: Univariate models and results of the tests

We have used a wholesale price index $P1$ and a cost of living index $P2$ published by the Instituto Nacional de Estadística (*Boletín del I.N.E.*). We have modelled these monthly series following the methods and computer package detailed in Hernández (1976), report here two equivalent models for each series.

Wholesale price index P1 (1965/1–1976/12)

$$(P1a) \quad \nabla \nabla_{12} (1 - 0.99B) \log P_t = (1 - 0.723B^{12})(1 - 0.941B)\varepsilon_t, \quad \sigma_\varepsilon^2 = 0.864 \times 10^{-4},$$

(0.0271) (0.068) (0.05) (0.00001)

$$(P1b) \quad \nabla \nabla_{12} \log P_t = (1 - 0.6893B^{12})(1 + 0.3118B^6)\varepsilon_t, \quad \sigma_\varepsilon^2 = 0.86 \times 10^{-4};$$

(0.072) (0.087) (0.00001)

Cost of living index P2 (1965/1–1976/12)

$$(P2a) \quad \nabla \nabla_8 \log P_t = (1 - 0.616B^8)(1 + 0.1308B)\varepsilon_t, \quad \sigma_\varepsilon^2 = 0.5 \times 10^{-4},$$

(0.079) (0.085) (0.000006)

$$(P2b) \quad (1 - 0.22B - 0.26B^8)(1 - 0.28B^{12})L_n P_t = \varepsilon_t + 0.0031, \quad \sigma_\varepsilon^2 = 0.5 \times 10^{-4}.$$

(0.08) (0.07) (0.09) (0.8 × 10⁻⁴)

Equivalence of these models is checked through their innovations cross-correlogram as illustrated in fig. 1.

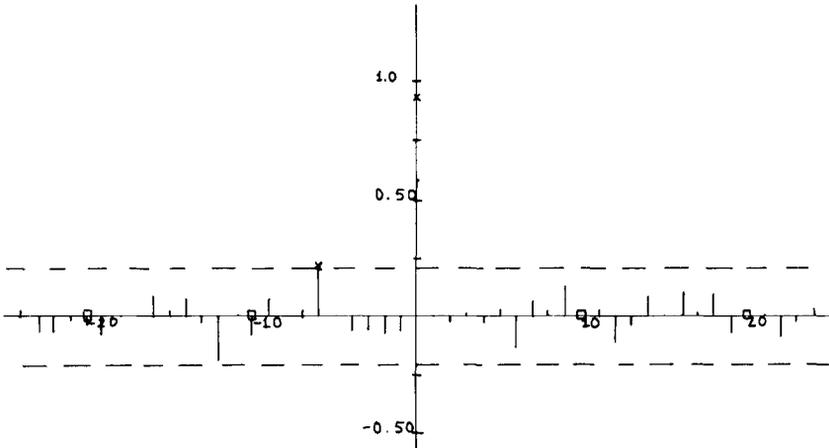


Fig. 1. Cross-correlations of innovations in models P1a and P1b, showing their equivalence.

In the case of the stock of money we have obtained models for $M1$, $M2$, and $M3$. The first two are published in the *Boletín Estadístico del Banco de España*, under the title ‘Oferta monetaria’, where $M1$ excludes time deposits and $M2$ is the stock including time deposits. $M3$ is the stock of money as published by the Instituto Nacional de Estadística also under the title ‘Oferta monetaria’. All series are published monthly, and this is the time period used here.

Oferta monetaria M1 (1967/1–1976/1)

$$(M1a) \quad \nabla \nabla_{12} \log M_t = (1 - 0.943B^{12})(1 - 0.3367B)\varepsilon_t, \quad \sigma_\varepsilon^2 = 0.55 \times 10^{-3},$$

(0.061) (0.096) (0.00008)

$$(M1b) \quad \nabla \nabla_{12}(1 + 0.27B)L_n M_t = (1 - 0.948B^{12})\varepsilon_t, \quad \sigma_\varepsilon^2 = 0.569 \times 10^{-3};$$

(0.098) (0.06) (0.00008)

Oferta monetaria M2 (1967/1–1976/1)

$$(M2a) \quad \nabla \nabla_{12} \ln M_t = (1 - 0.881B^{12})(1 - 0.3B)\varepsilon_t, \quad \sigma_\varepsilon^2 = 0.297 \times 10^{-3},$$

(0.06) (0.09) (0.00004)

$$(M2b) \quad \nabla \nabla_{12}(1 - 0.27B) \ln M_t = (1 - 0.89B^{12})\varepsilon_t, \quad \sigma_\varepsilon^2 = 0.3 \times 10^{-3};$$

(0.097) (0.05) (0.434 \times 10^{-4})

Oferta monetaria M3 (1967/1–1976/1)

$$(M3a) \quad \nabla \nabla_3 \nabla_{12} \ln M_t = (1 - 0.602B^{12})(1 - 0.646B^{-3})\varepsilon_t, \quad \sigma_\varepsilon^2 = 0.79 \times 10^{-4},$$

(0.074) (0.0688) (0.995 \times 10^{-5})

$$(M3b) \quad \nabla \nabla_3 \nabla_{12} \ln M_t = (1 - 0.59B^{12})(1 - 0.655B^3)(1 - 0.142B)\varepsilon_t, \quad \sigma_\varepsilon^2 = 0.778 \times 10^{-4}.$$

(0.075) (0.069) (0.09) (0.97 \times 10^{-5})

Figs. 2 to 6 show the cross-correlograms for these model innovations. We see from them that the Spanish data confirm the lack of causality, in the sense of Granger, that Feige and Pearce (1976) obtained for the American economy.

5.2. Interpretation of the results: The independence phenomenon

According to the model proposed in section 4, the innovations should satisfy eq. (15),

$$\varepsilon_x = \frac{(1-\lambda)}{(1-\beta B)} \varepsilon_y + \frac{(1-B)}{(1-\beta B)} e_1.$$

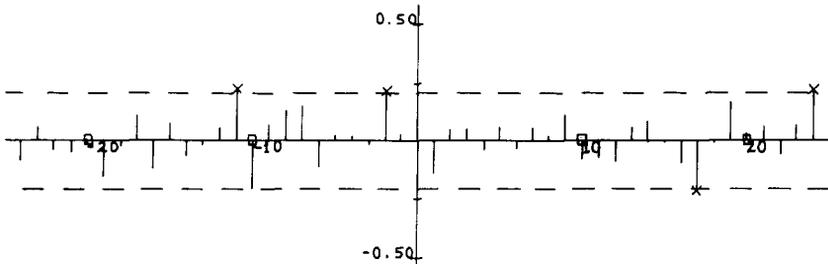


Fig. 2. Cross-correlations between series P1 and M1, using innovations of P1a and M1a.

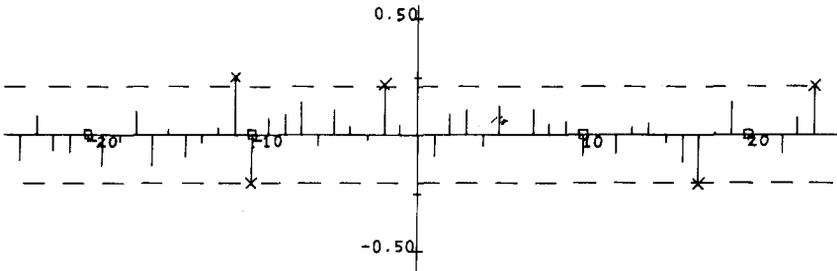


Fig. 3. Cross-correlations between series P1 and M2, using innovations of P1a and M2a.

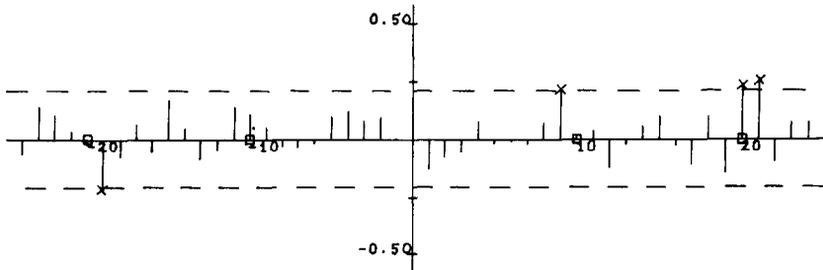


Fig. 4. Cross-correlations between series P2 and M1, using innovations of P2a and M1a.

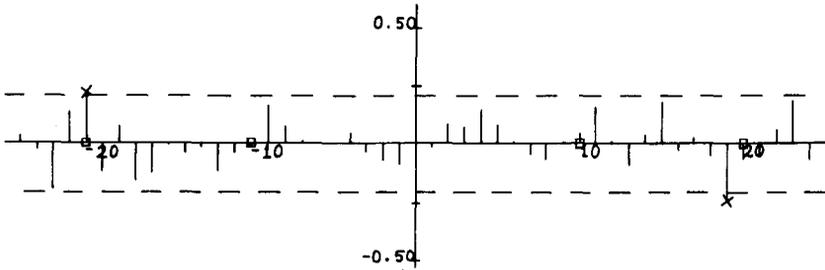


Fig. 5. Cross-correlations between series P2 and M2, using innovations of P2a and M2a.

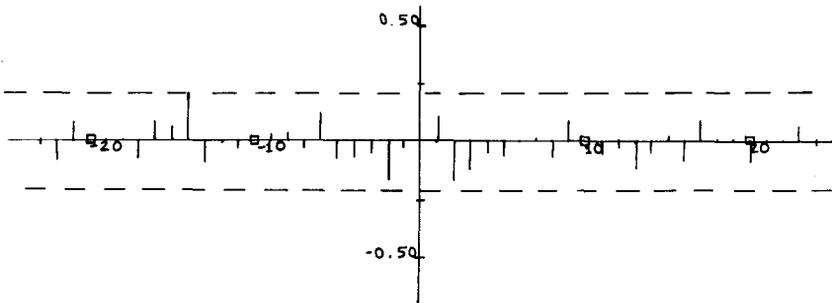


Fig. 6. Cross-correlations between series P2 and M3, using innovations of P2a and M3a.

The cross-correlograms should show the triangular structure of the model, producing unidirectional causality from prices to money that we have named ‘inverse causality’. However, as the results show independence, it might be thought that there is some conflict between the empirical evidence and the model. A closer look at the statistical implications of the model reveals that an interesting detectability problem crops up, and explains the eventual conflict.

In fact the largest of the cross-correlation coefficients would be

$$r_{\varepsilon_x \varepsilon_y}(0) = (1 - \lambda) \frac{\sigma_{\varepsilon_y}}{\sigma_{\varepsilon_x}}. \tag{16}$$

Therefore, if λ is close to unity we would get independence. How close? Bearing in mind the number of data used, and under the hypothesis of independence, the limits of acceptance of the null hypothesis correspond to a given critical value of λ . For $n = 120$ this value is 0.82 if $\sigma_{\varepsilon_y} \leq \sigma_{\varepsilon_x}$.

In the models for prices, for instance in P1a, we have estimated a value of $\lambda = 0.94$, which proves that the observed independence is compatible with a stable demand function for money.

We have estimated the Box–Jenkins univariate models of the U.S. time series and found $\lambda=0.81$ according to the model

$$\nabla^2 \ln P_t = (1 - 0.81B)\varepsilon_t, \quad \sigma_\varepsilon^2 = 0.477 \times 10^{-5}.$$

(0.037) (0.4 \times 10^{-6})

Feige and Pearce found $\lambda=0.84$ so that, if our monetary model was correct, independence between money and prices is the most likely outcome of causality analysis, in the sense of Granger. With moderate rates of inflation, both the U.S. and the Spanish economies are represented by a model where there is a stable demand function for money. But the inertia in the supply mechanism induces a very long-memory model of expectations which explains the spurious independence observed in the causality tests. Thus, because the actual estimated value of λ is very high, the observed lack of dependence does not refute the Quantity Theory, but rather confirms it.

The normal functioning of our economies provides a poor experiment [Pierce (1977)] to measure the parameters of the demand function and to directly determine the true causality from money to prices. For this reason we have tried to enrich our experiment by applying the same causality tests to Cagan's data for the German inflationary period. If the supply mechanism during this period did not correspond to the particular function that makes Cagan's expectations rational, one would expect bidirectional causality. We have modelled the rate of inflation $\log(P_t/P_{t-1})$ taken from Cagan (1956) and a money stock series taken from Graham (1930). The estimated models are:

Money stock (1918/6–1923/4)

$$\nabla V_3(1 - 0.917B) \log M_t = (1 - 0.9B^3)\varepsilon_t, \quad \sigma_{\varepsilon_t}^2 = 0.57 \times 10^{-3},$$

(0.05) (0.072) (0.0001)

Rate of inflation (1917/8–1923/4)

$$\nabla_3 \log(P_t/P_{t-1}) = (1 - 0.472B^3)(1 - 0.456B)\varepsilon_t, \quad \sigma_{\varepsilon_t}^2 = 0.446 \times 10^{-2}.$$

(0.139) (0.111) (0.0007)

Figs. 7 and 8 show the need for a three-months component. Fig. 9 shows the innovation cross-correlogram. We can in fact see that, for these series, we get bidirectional causality.

The model used to explain the relations between money and prices was proposed by Sargent (1977) to interpret Cagan's data. It is a correct model for periods of moderate rate of inflation. His estimated value for λ is much lower than ours. At this stage of the analysis the disparity should not be a surprise, since the model is not appropriate for an hyperinflationary period.

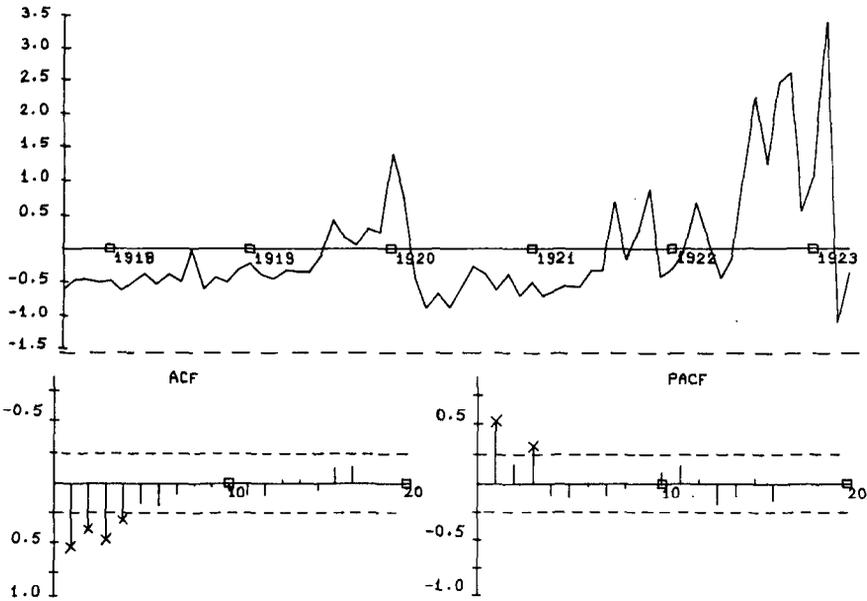


Fig. 7. $\log(P_t/P_{t-1})$, autocorrelation function and partial autocorrelation function for the period 1917/9-1923/4 in Germany.

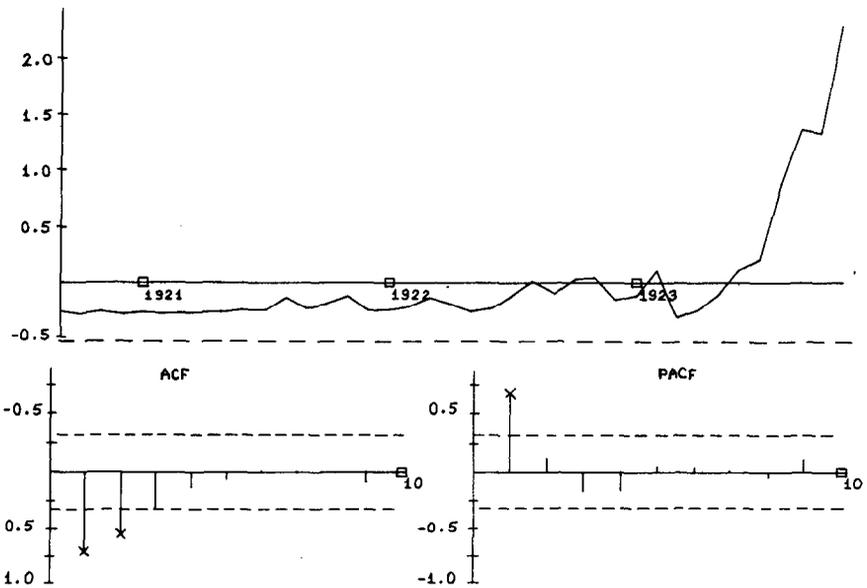


Fig. 8. $\log(P_t/P_{t-1})$, autocorrelation function and partial autocorrelation function for the period 1920/9-1923/11 in Germany.

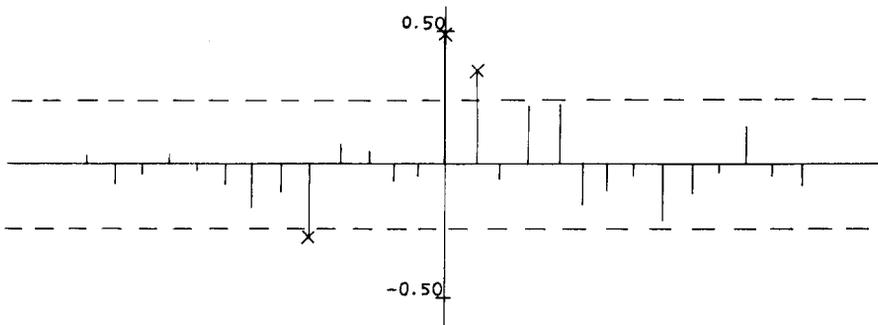


Fig. 9. Cross-correlations between series $\log(P_t/P_{t-1})$ and $\log M_t$ for German hyperinflation.

Sargent himself suspected that his model was not adequate when he said (1977, p. 78): 'Notice that my estimates of λ are always lower than Cagan's. That is an unexpected result, since according to the model, Cagan's estimate of λ and my maximum likelihood estimator are each consistent. The systematic difference in estimates ... may reflect the inadequacy of the model.' Thus the low estimate of the parameter λ by Sargent must be due to a misspecification of the hyperinflationary process. The model of expectations about the rate of inflation is wrong, since according to the causality results for this period, it must include information about the rate of money creation. A purely extrapolative model of these expectations is, in such a case, irrational in the sense of Muth. That the adequate model for the hyperinflation period must produce bidirectional causality is also supported by Frenkel's (1977) results.

It is clear, that the non-observability of causality from money to prices is only due to a feedback in the supply, which is not known to the public. On the other hand, absence of causality from prices to money is due to a detectability problem induced by the high value of the parameter λ . According to our model, the revealed behaviour of the monetary authorities (or the economic system reacting spontaneously in this way) is to keep a constant level of real money stock, acting with a very slow response. This slow response, in turn, generates a sluggish expectations mechanism, even if the desired cash balances are always assumed to adjust instantaneously to the existing stock.

Before closing this section, it is perhaps convenient to question how realistic our model is. Is it a mere 'curiosum', a simple rationalization of the available evidence? We think very much the contrary; it is a relevant description of the actual working of our economies under moderate rates of inflation. Here is not the right place to discuss the matter at length, but as Mints (1945) has shown the ancient 'real bills' doctrine has very often inspired monetary policy. As far as the Spanish economy is concerned, we

know that a variety of cost-push views of inflation have prevailed among the monetary authorities and still inspire thinking about inflation. That this implies feedback from prices to money, is clear in the following remark from one of the most influential Spanish monetary advisers, Rojo (1977): '... high rates of growth in the money stock represent a passive adaptation of the authorities to rapid growth in prices of a non-monetary origin.' If this is true, money does not have predictive value to anticipate the rates of inflation, and there is no surprise in the findings of causality analysis.

Many supply mechanisms, besides that estimated by us, may be relevant in other sample periods and still produce rational expectations of the rate of inflation which are purely extrapolative, i.e., correspond to univariate models. For instance, even a radical change of policy from a real-bills system to a Friedman supply rule, $x_t = k + e_{1t}$, would also result in independence. Therefore, we have to disagree with Feige and Pearce when they claim (1976, p. 519) that the independence results are a 'disturbing bit of evidence which appears to be in direct conflict with both popular doctrine and a substantial body of published econometric literature.'

6. Conclusions

The main conclusion of this paper is that the so-called independence phenomenon is compatible with the existence of a stable demand for money as formulated by the Quantity Theory. Cagan's model of expectations is inadequate for the German hyperinflation data that he studied, but represents correctly the expectations of economic agents in periods of moderate rates of growth in prices. The same law of demand originates different expectations under different regimes of money supply.

From Granger's independence result we have derived testable restrictions for a proposed model. The empirical evidence from the Spanish economy (similar to that of the American economy) allows us to estimate the parameters of the feedback mechanism that makes money endogenous, but we cannot estimate those of the demand.

Causality, as implied by the stable demand law, does not show up through the causality test. This lack of observability is due to what we have called 'inverse causality'. We have shown that inverse causality is also responsible for the observability problems in studies by Sargent, Nelson and McCallum (1979) and others. It is reasonably true that there are some important economic laws which are equally unobservable under the normal functioning of most economies. If this is so, there is a strong implication for macroeconomic modelling: the observed coincidence of efficient extrapolative and rational expectations is more the rule than the exception for most of the available historical samples.

The economic system usually provides little evidence of the causal effects implied by relevant economic laws: 'The economy is a miserable experimental design' [Pierce (1977, p. 20)]. For this reason theory is needed to look for those singular circumstances under which we can detect the causal relationships under study. As Poincaré (1909) said: 'Science after all is a matter of wise choice.'

Our work is in the spirit of Zellner and Palm (1974) who used time series analysis to test simultaneous equations models. We claim that Granger's causality tests in general and independence results in particular, far from being uninformative, provide testable restrictions to screen and refute alternative structural relationships.

We have illustrated the difference between causality according to a law [Zellner (1978)] and causality in the sense of Granger. We pointed out the need for further interplay between economic theory and statistical work. Once more, it is clear that there are good reasons to make ours the words of Marschak (1950): '*Thus, practice requires theory.*'

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