DETERMINING THE ORDER-UP-TO-LEVEL USING THE NORMAL APPROXIMATION IN A DISCRETE CONTEXT

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AGENDA

- Introduction
- The Traditional fill rate expression in periodic systems
- Experimental design
- Analysis of the results
- Conclusions and practical implications
INTRODUCTION

Inventory Management

When a replenishment order should be placed?

How much should be ordered?
INTRODUCTION

Inventory Management

When a replenishment order should be placed?

How much should be ordered?

Periodic Review $(R, S)$

Review Period, $R$ (predefine)

Order-up-to-Level, $S$
INTRODUCTION

Inventory Management

When a replenishment order should be placed?

Periodic Review ($R, S$)

Review Period, $R$
(predefine)

How much should be ordered?

Order-up-to-Level, $S$

Min costs

Target service level

Fill Rate, $FR$
INTRODUCTION

Inventory Management

**When a replenishment order should be placed?**

**Order-up-to-Level, S**

**Review Period, R** (predefine)

**Min costs**

**Target service level**

**Fill Rate, \( FR \)**

\( FR_{Trad} \sim \) Normal distribution
**INTRODUCTION**

**Inventory Management**

- **When a replenishment order should be placed?**
- **How much should be ordered?**

**Periodic Review** $(R, S)$

- **Review Period,** $R$ (predefine)
- **Order-up-to-Level,** $S$

**Target service level**

Min costs

**Fill Rate,** $FR$

$FR_{Trad} \sim \text{Normal distribution}$
INTRODUCTION

**Inventory Management**

- **When** place a replenishment order?
- **How much** should be ordered?

**Periodic Review (R, S)**

- **Review Period, R** (predefine)
- **Order-up-to-Level, S**

**Purpose**

Determining under which conditions the $FR_{Trad}$ provides an accurate enough estimation of the $S$ in a discrete demand context.

$FR_{Trad} \sim$ Normal distribution
THE TRADITIONAL \( FR \) IN PERIODIC SYSTEMS

**Fill Rate, \( FR \)**

- Fraction of demand that can be immediately served using the on hand stock (Lee and Billington, 1992)

\[
FR = 1 - \frac{E(\text{unfulfilled demand})}{E(\text{total demand})} \quad \text{or} \quad FR = \frac{E(\text{fulfilled demand})}{E(\text{total demand})}
\]
The periodic review inventory system

THE TRADITIONAL FR IN PERIODIC SYSTEMS

The periodic review inventory system

On hand stock ($OH$)
Inventory position ($IP$)
Net stock ($NS$)

Review
Arrival
Review
Arrival
The periodic review inventory system

Key time period: \( R+L \)

Units

On hand stock \((OH)\)

Inventory position \((IP)\)

Net stock \((NS)\)

Review

Arrival

Review

Arrival

Key time period: \( R+L \)
Traditional Fill Rate, $FR_{Trad}$

- (Hadley and Whitin, 1963) derive a simple approximation to compute the expected shortage as:

$$E(\text{shortage}) = E\left[D_{R+L} - S\right]^+$$

- Considering Normal demand distribution with mean $\mu$ and variance $\sigma^2$

$$E(\text{shortage}) = \sigma_{R+L} G\left(k_{R+L}(S)\right)$$

where $G(k_t(S))$ is a special function of the safety stock factor $k_t(S)$ and the standard normal distribution (Appendix B of (Silver et al., 1998))
Traditional Fill Rate, $FR_{Trad}$

- In a backorder system

\[ E(\text{shortage}) = E(\text{unfulfilled demand}) \]

\[ FR = 1 - \frac{E(\text{unfulfilled demand})}{E(\text{total demand})} \]

Then, the fill rate can be computed following the (Hadley and Whitin, 1963) expression as:

\[ FR = 1 - \frac{E(\text{shortage})}{E(\text{total demand})} \]

\[ FR = 1 - \frac{\sigma_{R+L} G(k_{R+L}(S))}{R \mu} \]
**Purpose**

- The estimation of the parameters of a periodic review \((R, S)\) policy when the \(FR_{Trad}\) is used and demand is discrete

- The problem consists of calculating the smallest order-up-to-level \(S\) that guarantees the achievement of the \(FR^*\)

**Data**

<table>
<thead>
<tr>
<th>Factor</th>
<th>Values</th>
<th>Total cases: 235,620</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demand distribution</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poisson (\lambda)</td>
<td>0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.75, 0.9, 1, 1.25, 1.5, 1.75, 2, 2.5, 3, 4, 5, 7, 10, 15, 20</td>
<td></td>
</tr>
<tr>
<td>Binomial (n)</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 15, 20</td>
<td></td>
</tr>
<tr>
<td>(\theta)</td>
<td>0.01, 0.05, 0.1, 0.15, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99</td>
<td></td>
</tr>
<tr>
<td>Negative Binomial (r)</td>
<td>0.05, 0.1, 0.2, 0.25, 0.3, 0.4, 0.5, 0.75, 0.9, 1, 1.25, 1.5, 1.75, 2, 2.5, 3, 3.5, 4</td>
<td></td>
</tr>
<tr>
<td>(\theta)</td>
<td>0.1, 0.15, 0.25, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99</td>
<td></td>
</tr>
<tr>
<td><strong>Inventory System</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(FR^*)</td>
<td>0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95, 0.99</td>
<td></td>
</tr>
<tr>
<td>(R)</td>
<td>1, 2, 3, 4, 5, 7, 10, 15, 20</td>
<td></td>
</tr>
<tr>
<td>(L)</td>
<td>1, 3, 5, 7, 10, 15, 20</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Experimental factors and values
Assumptions

- $L$ is constant
- predefined review period $R$
- excess demand is backordered
- the replenishment order is added to the inventory at the end of the period in which it is received
- demand during a period is fulfilled with the on hand stock at the beginning of the period
- the demand process is assumed to be stationary with a known, discrete and i.i.d. distribution function
EXPERIMENTAL DESIGN

Experimental Design

STEP 1
Combination of a set of parameters of $R$, $L$, demand and $FR^*$

STEP 2
Calculation of minimum $S$ with the Traditional $FR$ ($S_{\text{Trad}}$)

STEP 3
Calculation of minimum $S$ with the Exact $FR$ ($S_{\text{Exact}}$)

STEP 4
Analyzing and quantifying deviations
**Experimental Design**

**STEP 1**
Combination of a set of parameters of $R$, $L$, demand and $FR^*$

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Analyzing and quantifying deviations

Relative errors ($RE$)

$$RE = \frac{S_{Exact} - S_{Trad}}{S_{Exact}}$$
**Experimental Design**

**STEP 1**
Combination of a set of parameters of $R$, $L$, demand and $FR^*$

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**STEP 3**
Calculation of minimum $S$ with the Exact FR ($S_{Exact}$)

**STEP 4**
Analyzing and quantifying deviations

**relative errors ($RE$)**

$$RE = \frac{S_{Exact} - S_{Trad}}{S_{Exact}}$$

- If $RE < 0$
  - $S_{Trad} > S_{Exact} \Rightarrow$ The system *reaches the target fill rate*, although increasing the average stock level and the holding costs

- If $RE > 0$
  - $S_{Trad} < S_{Exact} \Rightarrow$ The policy is *not reaching the target fill rate* and therefore the system is *less protected* against stockouts than managers believe.
RESULTS ANALYSIS

Classification and Regression Trees (CART)

- Data mining technique → exploratory tool to identify under which conditions (*demand pattern and inventory characteristics*) the $FR_{Trad}$ is accurate enough to estimate $S$ in a *discrete* demand context
RESULTS ANALYSIS

 Classification and Regression Trees (CART)

- Data mining technique → exploratory tool to identify under which conditions (demand pattern and inventory characteristics) the FR_{Trad} is accurate enough to estimate S in a discrete demand context.

 Definition of variables

- Dependent variable: categorical variable

  "yes" if \( S_{Trad} = S_{Exact} \)
  
  "no" if \( S_{Trad} \neq S_{Exact} \)
RESULTS ANALYSIS

Classification and Regression Trees (CART)

- Data mining technique → exploratory tool to identify under which conditions (demand pattern and inventory characteristics) the $FR_{Trad}$ is accurate enough to estimate $S$ in a discrete demand context

Definition of variables

- **Dependent variable**: categorical variable
  
  "yes" if $S_{Trad} = S_{Exact}$
  
  "no" if $S_{Trad} \neq S_{Exact}$

- **Independent variables**:
  
  - categories of demand (erratic, lumpy, smooth and intermittent)
  - squared coefficient of variation of demand sizes, $CV^2$
  - average inter-demand interval, $p$
  - the target fill rate, $FRO$
  - the mean of the random variable during $R$, $\mu_R$
  - mean of the random variable during $R+L$, $\mu_{R+L}$
  - exact $S$, $S_{Exact}$
RESULTS ANALYSIS
RESULTS ANALYSIS

Good performance

If CV² ≤ 0.306

Bad performance

If CV² > 0.306
RESULTS ANALYSIS

Category of demand
Syntetos et al., 2005

Good performance

Bad performance
RESULTS ANALYSIS

There are some cases where $S_{\text{Trad}} \neq S_{\text{Exact}}$.

Bad performance

Good performance
RESULTS ANALYSIS

Classification errors

- Knowing the classification errors that imply choosing the $FR_{Trad}$ in every single node, even if the exact method is more efficient.

- Table 2 presents the percentage of misclassified cases per node, the typology of the errors (RE1 or RE2) and its mean and variance.

<table>
<thead>
<tr>
<th>Node identification</th>
<th>% cases $RE = 0$</th>
<th>% $RE1$</th>
<th>$\mu_{RE1}$ (%)</th>
<th>$\sigma_{RE1}$ (%)</th>
<th>% $RE2$</th>
<th>$\mu_{RE2}$ (%)</th>
<th>$\sigma_{RE2}$ (%)</th>
</tr>
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<tbody>
<tr>
<td>CV2≤0.306 and $p$≤1.367</td>
<td>85,5</td>
<td>11,5</td>
<td>-10,6</td>
<td>16,2</td>
<td>2,9</td>
<td>6,9</td>
<td>8,9</td>
</tr>
<tr>
<td>CV2≤0.306; $p$&gt;1.367 $FRO$≤0.97 and $S$≤29</td>
<td>72,1</td>
<td>22,0</td>
<td>-35,1</td>
<td>30,9</td>
<td>5,9</td>
<td>29,8</td>
<td>16,2</td>
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<tr>
<td>CV2≤0.306; $p$&gt;1.367 $FRO$≤0.97 and $S$&gt;29</td>
<td>40,7</td>
<td>48,2</td>
<td>-3,6</td>
<td>4,1</td>
<td>11,1</td>
<td>1,8</td>
<td>1,5</td>
</tr>
<tr>
<td>CV2≤0.306; $p$&gt;1.367 and $FRO$&gt;0.97</td>
<td>21,8</td>
<td>0,2</td>
<td>-14,7</td>
<td>11,8</td>
<td>78,0</td>
<td>19,3</td>
<td>15,8</td>
</tr>
<tr>
<td>CV2 &gt; 0.306</td>
<td>33,5</td>
<td>43,5</td>
<td>-13,7</td>
<td>15,2</td>
<td>23,0</td>
<td>11,2</td>
<td>11,1</td>
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Table 2. Misclassified errors per node, identifying the typology or error (RE1 or RE2)
RESULTS ANALYSIS

Classification errors

- Knowing the classification errors that imply choosing the $FR_{Trad}$ in every single node, even if the exact method is more efficient.

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| $CV2 \leq 0.306$; $p > 1.367$  
$FRO \leq 0.97$ and $S \leq 29$ | 72,1 | 22,0 | -35,1 | 30,9 | 5,9 | 29,8 | 16,2 |
| $CV2 \leq 0.306$; $p > 1.367$  
$FRO \leq 0.97$ and $S > 29$ | 40,7 | 48,2 | -3,6 | 4,1 | 11,1 | 1,8 | 1,5 |
| $CV2 \leq 0.306$; $p > 1.367$  
and $FRO > 0.97$ | 21,8 | 0,2 | -14,7 | 11,8 | 78,0 | 19,3 | 15,8 |
| $CV2 > 0.306$ | 33,5 | 43,5 | -13,7 | 15,2 | 23,0 | 11,2 | 11,1 |

Table 2. Misclassified errors per node, identifying the typology or error (RE1 or RE2)
CONCLUSIONS

Information of Table 2

Twofold purpose:

- **Decision tool** → If managers know the *characteristics of the item*, they can know if $FR_{Trad}$ is an *appropriate tool* to establish the $S$ or they should use the exact expression.

- **Corrective tool** → if a company is using the $FR_{Trad}$ to determine the $S$, the results of this work provide information about the *risk* of using it.

It seems to be a *relationship* between the *category of the demand* and the *performance* of $FR_{Trad}$. 
CONCLUSIONS

Erratic

Lumpy

Smooth

Intermitent

Syntentos et al., 2005

CV²

1.32

0.49

p
CONCLUSIONS

Node 3

CV\(^2\) vs. \(p\)

Erratic

Lumpy

Smooth

Intermitent

Bad performance

1.32

0.49
CONCLUSIONS

Node 3

Erratic
Lumpy

Bad performance

Node 4

Smooth
Intermitent

85.5% cases $S_{Trad} = S_{Exact}$
CONCLUSIONS

Node 3

- Erratic
- Lumpy

Node 4

- Smooth
- Intermittent

Nodes 8, 9, 7

- 85.5% cases $S_{\text{Trad}} = S_{\text{Exact}}$
- Depends on $FR^*$ and $S$

- If $FR^* > 0.97 \rightarrow \text{bad performance}$
CONCLUSIONS

Further research

- Studying other fill rate expressions and proposing a reference framework to choose the most efficient method to compute the order-up-to-level

- Extending these results to the lost sales context
Thank you for your the attention

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