Applications of the Lagrangian Relaxation Method to Operations Scheduling

Laviós-Villahoz J.J [1],Arauzo J.A[2], del-Olmo-Mártinez R[3], Manzanedo-del-Campo M.A[4],

Abstract Lagrangian Relaxation is a combinatorial optimization method, which is mainly used as decomposition method, so a complex problem is divided into smaller and easier problems. Lagrangian Relaxation method has been applied to solve scheduling problem in diverse manufacturing environments like single machine, parallel machine, flow shop, job shop or even in complex real-world environments. We highlight two key issues on the application of the method: the first one is about the method to solve the dual problem and second one is the choice of the constraints to relax. We present the main characteristics of these approaches and survey the existing works in this area.

Keywords: Lagrangian Relaxation, Scheduling, Combinatorial Optimization, Integer Programming

[1]Juan José Lavios Villahoz (✉ e-mail: jjlavios@ubu.es)
Grupo INSISOC. EPS. Universidad de Burgos. Av. Cantabria s/n, 09006, Burgos

[2]José Alberto Arauzo Arauzo (✉ e-mail: arauzo@insisoc.org)
Grupo INSISOC. EII. Universidad de Valladolid. Pso del Cauce 59, 47011 Valladolid.

[3]Ricardo del Olmo Martínez (✉ e-mail: rdelolmo@ubu.es)
Grupo INSISOC. EPS. Universidad de Burgos. C/ Villadiego s/n, 09001, Burgos.

[4]Miguel Ángel Manzanedo del Campo (✉ e-mail: mmanz@ubu.es)
EPS. Universidad de Burgos. C/ Villadiego s/n, 09001, Burgos.

* The authors acknowledge support from the Spanish Ministry of Science and Innovation Project CSD2010-00034 (SimulPast CONSOLIDER-INGENIO 2010) and by the Regional Government of Castile and Leon (Spain) Project VA056A12-2.

1 Introduction

The Lagrangian Relaxation method seeks the solutions of a complex optimization problem from the solutions of an easier problem, which is obtained by relaxing the complex constraints of the original problem. It is mainly used as a decomposition method, where complex problems are divided into smaller and easier problems. It has been applied to solve scheduling problems in diverse manufacturing environments like single machine, parallel machine, flow shop, job shop or even in complex real-world environments. The two key points in Lagrangian Relaxation are to decide which constraints are relaxed and to choose which method should be used to solve the dual problem. These decisions determine the performance of the method.

The rest of the paper is organized as follows: in section 2 we introduce the basic idea of Lagrangian Relaxation and discuss two of the main choices in the design of the method, which constraints to relax and which method apply to solve the dual problem; in section 3, we review the main works that apply the Lagrangian Relaxation to the scheduling problem in a manufacturing environment; in section 4, the conclusions are presented.

2 Implementation of the Lagrangian Relaxation in a Manufacturing Scheduling Problem

The basic idea of Lagrangian Relaxation is to remove some of the constraints and add them to the objective function. Each of the relaxed constraints is added to the objective function multiplied by a variable (Lagrange multiplier) that penalize the violation of the constraints. The key is to relax the constraints such that the relaxed problem is easier than the original problem.

The Lagrangian Relaxation method has been applied to different scheduling environments like single machine, parallel machine, flow shop, job shop, real-world environments, projects and supply chain. Integer formulation can be used to formulate these problems. Total weighted tardiness or total weighted earliness and tardiness is used as objective function. As additive functions, they can be easily decomposed into job dependent subfunctions.

In a manufacturing environment machine capacity constraints and task precedence constraints are the main candidates to be relaxed. The relaxed constraints will determinate the orientation of the problem decomposition. If capacity constraints are relaxed, the problem can be decomposed into job related subproblems, but if precedence constraints are relaxed, the problem can be decomposed into machine related subproblems. Wang et al (1997) and Chen et al (1998) indicates that the relaxation of precedence constraints causes important oscillations in the solution from iteration to iteration and prevent convergence of the algorithm.

When the number of machines is large enough the relaxation of the capacity constraints leads to better lower bounds (Baptiste et al 2008).

The Lagrangian Relaxation method transforms the solutions of the relaxed problem into feasible solutions of the original problem, trying to obtain near-optimal solutions. The optimal value of the relaxed problem is a lower bound of the optimal objective of the original problem. The subgradient method iteratively adjusts the Lagrangian multipliers to find the best lower bound of the optimal objective value of the original problem (Fisher 2004). However, the subgradient method requires the optimization of all the subproblems. It has two main drawbacks: first, in some environments it may be difficult to obtain the optimum of the relaxed problem; and second, the optimization of large size problems can be very time consuming.

To overcome these difficulties the surrogate subgradient method have been proposed. The main advantage is that the minimization of all the subproblems is not required. In this method only a near optimization is sufficient to converge, so less computational effort is needed (Zhao et al 1997). The main drawback of this method is that the updating direction is worse than in the subgradient method.

As is shown in Table 1 most researches relax capacity constraints rather than precedence constraints and the surrogate subgradient method is preferred for complex environments or in large problems.

Table 1 Problem environment, dual problem method and type of constraints relaxed

	subgradient method		surrogate gradient method	
Relaxed constraints	Capacity constraints	Precedence constraints	Capacity constraints	Precedence constraints
General scheduling problem	Luh and Hoitomt (1993)			
Single machine	Sun et al (1999); Jeong and Leon (2005); Tang et al (2007);	Jiang and Tang (2008)		
Parallel machine	Luh et al (1990); Edis et al (2008); Tang and Zhang (2009)			
Flow shop	Nishi et al 2007)	Tang et al (2006)		
Job shop	Chen et al (1995, 1998); Wang et al (1997); Dewan and Joshi (2002); Luh and Feng (2003); Liu et al (2004, 2007); Baptiste et al (2008); Jeong and Yim (2009)	Hoitomt et al (1993)	Wang et al (1997); Kaskavelis and Caramanis (1998); Zhao et al (1999);	Chen and Luh (2003)

Table 1 (continued) Problem environment, dual problem method and type of constraints relaxed

	subgradient method		surrogate gradient method	
Relaxed constraints	Capacity constraints	Precedence constraints	Capacity constraints	Precedence constraints
Complex environment	Masin et al (2007); Arauzo (2007) Arauzo et al (2009)			Zhang et al (2001); Chen et al (2003); Luh et al (2003), Sun et al (2006)

3 Applications of the Lagrangian Relaxation Method to Different Manufacturing Environments.

In this section, we review the main works that apply the Lagrangian Relaxation to the scheduling manufacturing problems, they are classified by the type of environment.

General scheduling problem

Luh and Hoitomt (1993) propose a generic methodology that uses Lagrangian Relaxation to solve a scheduling problem. It is applied to three different scheduling environments: individual operations to be scheduled in identical parallel machines; job orders consisting of multiple operations to be scheduled in identical parallel machines; and tasks related with general precedence constraints which have to be scheduled in different machines.

Single machine scheduling problem

Sun et al (1999) use Lagrangian Relaxation algorithm to solve the problem of scheduling a single machine with sequence dependent setup time. Sequence dependent setup times are modeled as capacity constraints.

Dewan and Joshi (2000) decomposes the problem into a local job problems and the machine problem. The machine fixes the prices of time slots and jobs chose the time slots that optimize their costs. This arises in a dynamic environment where new job orders may enter at any time. Every time a job is entered into the system the prices are recalculated, taking the last prices before the arrival of the new order as initial prices.

Jeong and Leon (2005) apply a variation of the dual problem resolution to solve the problem of scheduling a shared resource of three production subsystems without using a central planning system. They search a fully distributed problem.

Tang et al (2007) deal with multiple immediate predecessors and successors. A forward and backwards dynamic programming algorithm is developed to solve the relaxed problem.

Parallel machines

Luh et al (1990) first apply the Lagrangian Relaxation method to schedule jobs on several identical machines. They define an iterative algorithm. The feasible program is obtained from the task sequence extracted from the relaxed problem.

Edis et al (2008) add machine eligibility restrictions. In these problems different machines can do the same job with a different performance. They develop a heuristic to build feasible programs from the relaxed problem.

Tang and Zhang (2009) apply Lagrangian Relaxation method to rescheduling tasks on a parallel machine in a dynamic environment. Breakdown of machines is considered. The goal is to reschedule tasks with minimal modifications to the original problem, but taking into account the efficiency the obtained schedule.

Flow Shop problem

Lagrangian Relaxation is applied in Tang et al (2006) to the problem of scheduling a hybrid flow shop system. This system comprises various stages of production, and in each of these stages there are a number of machines working in parallel.

In Nishi et al (2007) the Lagrangian Relaxation algorithm is iteratively applied while new restrictions are progressively added. They make new proposals for improving the dynamic programming step.

Job shop problem

Hoitomt et al (1993) relax the precedence and the capacity constraints. A list-scheduling algorithm is developed to generate a feasible schedule.

The main contribution of Chen et al (1995) is the use of dynamic programming techniques to solve the relaxed job shop problem at the job order level. This avoids additional relaxation of the precedence constraints.

Wang et al (1997) minimize earliness and tardiness of job orders. They compare several methods for solving the dual problem.

Chen et al (1998) propose to relax only the capacity constraints to decompose the problem into several subproblems associated to job orders. These problems are solved using dynamic programming technique. They apply it to real factory data.

Kaskavelis and Caramanis (1998) and Zhao et al (1999) apply the surrogate dual method to a separable problem (interleaved subgradient method).

The main contribution of Dewan and Joshi (2002) is the study of the relationship between the theory of auctions and Lagrangian Relaxation. The concept of multi-agent systems is used as a basis for the implementation of distributed systems in planning and production control. They define the problem as combinatorial auctions where the items sold are the time slot in which the planning horizon is divided. Liu et al (2004, 2007) also define a reactive scheduling system where auctions are held in a machine whenever a time slot is free.

Chen and Luh (2003) solve the job shop problem using the Lagrangian Relaxation method and relaxing the precedence constraints, rather than capacity constraints. The resulting subproblems are equivalent to solving a problem of scheduling a single machine or parallel machines.

Kutanoglu and Wu (2004) combine the stochastic analysis with the dynamic adaptation of the system to improve the robustness of Lagrangian Relaxation method.

Baptiste et al.(2008) compare precedence constraints and capacity constraints. They improve the solutions by local search.

Jeong and Yim (2009) apply Lagrangian Relaxation to solve the job shop problem extended to the case of virtual enterprise with agents controlling one or more job orders and machines. Each subsystem calculates its own schedule. They use the protocol CICA, developed in Jeong and Leon (2002), as a framework to solve the scheduling problem in a distributed manner using an agent-based environment

Scheduling in complex environments

Gou et al (1998) implement the problem of production scheduling in a real production plant using a multi-agent system. They organize the system as a hierarchy following a quasi-distributed holonic structure based on relaxation of the capacity constraints and precedence of the job shop problem.

Zhang et al (2001) develop a macro-level scheduling method based on Lagrangian Relaxation method. Large problems are studied and assemblies and disassemblies are allowed. This makes the structure of the resolution process and the subproblems complex. Surrogate subgradient method is proposed to reduce the computation requirements.

Chen et al (2003) study a complex environment with multiple resources, setup times and transfer lots. They use a dynamic programming method for solving the subproblems related to transfer lots easier than in the previous work.

Luh et al (2003) apply Lagrangian Relaxation to a problem of supply chain as an extension of a job shop problem. It uses a variation of the Contract Net protocol for communication among agents and the surrogate gradient method to solve the dual problem.

Sun et al (2006) study complex process structures with coupling assemblies and disassemblies. They propose to relax only one of the precedence constraints to avoid oscillation problems. They add an auxiliary function penalty to improve its convergence.

Arauzo (2007) implements an auction mechanism for controlling flexible manufacturing systems. The analogy between auction and Lagrangian Relaxation process is applied in the definition of the auction. This provides a robust mathematical method in the updating prices step.

Arauzo et al (2009) propose a multi-agent system for project portfolio management. Projects negotiate the allocation of shared resources by an auction. The analogy between the Lagrangian Relaxation method and auctions is used as a modeling tool. This approach allows managing the company's current projects and provides decision criteria for the acceptance or rejection of new projects.

4 Conclusions

We reviewed the studies that have applied the Lagrangian Relaxation method to the resolution of scheduling operations in recent years. The scope has been diverse and covers most types of problems. The application of the method has been performed with two types of relaxation of constraints: capacity or precedence constraints. The choice of one or another implies respectively an approach towards a job decomposition or a machine decomposition of the problem. The dual problem has also been solved with two different methods: the subgradient method, the surrogate gradient method. The objectives of these studies aim to improve the speed of resolution, the stability of the method, and the ease of solving problems.

5 References

Arauzo JA (2007) Control distribuido de sistemas de fabricación flexibles: un enfoque basado en agentes. Thesis. University of Valladolid.

Araúzo JA, Pajares J, Lopez-Paredes A, Pavón J (2009) Agent-based modeling and simulation of multi-project scheduling. Multi-Agent Logics, Languages, and Organizations Federated Workshops.

Baptiste P, Flamini M, Sourd F (2008) Lagrangian bounds for just-in-time job-shop scheduling. Computers and Operations Research 35(3):906-915.

Chen H, Chu C, Proth J-M (1998) An improvement of the Lagrangean relaxation approach for job shop scheduling: a dynamic programming method. IEEE Transactions on Robotics and Automation 14(5):786-795.

Chen H, Chu C, Proth J-M (1995) More efficient Lagrangian relaxation approach to job-shop scheduling problems. In Proceedings of the IEEE International Conference on Robotics and Automation 496-501.

Chen H, Luh PB (2003) An alternative framework to Lagrangian relaxation approach for job shop scheduling. European Journal of Operational Research 149(3):499-512.

Chen D, Lut PB, Thakur LS, Moreno Jr, J (2003) Optimization-based manufacturing scheduling with multiple resources, setup requirements, and transfer lots. IIE Transactions 35(10):973-985.

Dewan P, Joshi S (2002) Auction-based distributed scheduling in a dynamic job shop environment. International Journal of Production Research 40(5):1173-1191.

Dewan P, Joshi S (2000) Dynamic single-machine scheduling under distributed decision-making. International Journal of Production Research 38(16):3759-3777.

Edis EB, Araz C, Ozkarahan I (2008) Lagrangian-based solution approaches for a resource-constrained parallel machine scheduling problem with machine eligibility restrictions. In *Proceedings of the 21st international conference on Industrial, Engineering and Other Applications of Applied Intelligent Systems: New Frontiers in Applied Artificial Intelligence*. 337-346. Springer, Wrocław.

Fisher ML (2004) The Lagrangian relaxation method for solving integer programming problems. Management Science 50(12_supplement):1861-1871.

Gou L, Luh PB, Kyoya Y (1998) Holonic manufacturing scheduling: architecture, cooperation mechanism, and implementation. Computers in Industry 37:213-231.

Hoitomt DJ, Luh PB, Pattipati KR (1993) A practical approach to job-shop scheduling problems. IEEE Transactions on Robotics and Automation 9(1):1-13.

Jeong I-J, Leon VJ (2005) A single-machine distributed scheduling methodology using cooperative interaction via coupling agents. IIE Transactions 37(2):137-152.

Jeong I-J, Leon VJ (2002) Decision-making and cooperative interaction via coupling agents in organizationally distributed systems. *IIE Transactions* 34(9):789-802.

Jeong I-J, Yim S-B (2009) A job shop distributed scheduling based on Lagrangian relaxation to minimise total completion time. International Journal of Production Research 47(24):6783.

Jiang S, Tang L (2008) Lagrangian relaxation algorithm for a single machine scheduling with release dates. In Second International Symposium on Intelligent Information Technology Application 811-815.

Kaskavelis CA, Caramanis MC (1998) Efficient Lagrangian relaxation algorithms for industry size job-shop scheduling problems. IIE Transactions 30(11):1085-1097.

Kutanoglu E, Wu SD (2004) Improving scheduling robustness via preprocessing and dynamic adaptation. IIE Transactions 36(11):1107.

Liu N, Abdelrahman MA, Ramaswamy S (2007) A complete multiagent framework for robust and adaptable dynamic job shop scheduling. IEEE Transactions on Systems, Man and Cybernetics Part C: Applications and Reviews 37(5):904-916.

Liu N, Abdelrahman MA, Ramaswamy S (2004) A multi-agent model for reactive job shop scheduling. In Proceedings of the Thirty-Sixth Southeastern Symposium on System Theory 241-245.

Luh PB, Feng W (2003) From manufacturing scheduling to supply chain coordination: The control of complexity and uncertainty. Journal of Systems Science and Systems Engineering 12(3):279-297.

Luh PB, Hoitomt DJ (1993) Scheduling of manufacturing systems using the Lagrangian relaxation technique. IEEE Transactions on Automatic Control 38(7):1066-1079.

Luh PB, Ni M, Chen, H, Thakur LS (2003) Price-based approach for activity coordination in a supply network. IEEE Transactions on Robotics and Automation 19(2):335-346.

Luh PB, Omt D, Max E, Pattipati KR (1990) Schedule generation and reconfiguration for parallel machines. IEEE Transactions on Robotics and Automation 6(6):687-696.

Masi M, Pasaogullari MO, Joshi S (2007) Dynamic scheduling of production-assembly networks in a distributed environment. IIE Transactions 39(4):395-409.

Nishi T, Hiranaka Y, Inuiguchi M (2007) A successive Lagrangian relaxation method for solving flow shop scheduling problems with total weighted tardiness. In IEEE International Conference on Automation Science and Engineering 875-880.

Sun T, Luh PB, Min L (2006) Lagrangian relaxation for complex job shop scheduling. In Proceedings of the IEEE International Conference on Robotics and Automation 1432-1437.

Sun X, Noble JS, Klein CM (1999) Single-machine scheduling with sequence dependent setup to minimize total weighted squared tardiness. IIE Transactions 31(2):113-124.

Tang L, Zhang Y (2009) Parallel machine scheduling under the disruption of machine breakdown. Industrial and Engineering Chemistry Research 48(14):6660-6667.

Tang L, Xuan H, Liu J (2007) Hybrid backward and forward dynamic programming based Lagrangian relaxation for single machine scheduling. Computers and Operations Research 34(9):2625-2636.

Tang L, Xuan H, Liu J (2006) A new Lagrangian relaxation algorithm for hybrid flowshop scheduling to minimize total weighted completion time. Computers and Operations Research 33(11):3344-3359.

Wang J, Luh PB, Zhao X, Wang J (1997) An optimization-based algorithm for job shop scheduling. Sädhana 22(part 2):241-256.

Zhang Y, Lut PB, Narimatsu K et al (2001) A macro-level scheduling method using Lagrangian relaxation. IEEE Transactions on Robotics and Automation 17(1):70-79.

Zhao X. Luh PB, Wang J (1999) Surrogate gradient algorithm for Lagrangian relaxation. Journal of Optimization Theory and Applications 100(3):699-712.

Zhao X, Luh P, Wang J (1997) The surrogate gradient algorithm for Lagrangian relaxation method. Proceedings of the 36th IEEE Conference on Decision and Control 1:310, 305.